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Examination of teachers' noticing: the mathematical modeling process*

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Abstract

This study investigates the change in the mathematics teachers' noticing levels through modeling. The case study method obtained extensive, comprehensive, and in-depth data. The study used modeling activities, group activity solutions, semi-structured interview sheets, observation notes, and video and audio recordings as data collection tools. The framework developed by Estapa et al. (2018) was used to analyze the data and examine the ability of mathematics teachers to notice during the application process of mathematical modeling activities. In addition, van Es (2011) developed the framework to determine the level of teachers' noticing skills. Seven elementary school mathematics teachers participated in the study. As a result of the study, when the data were examined, it was found that the level of noticing skills of six of the seven teachers increased during mathematical modeling activities. At the beginning of the study, the teachers did not allow the students to understand and interpret the question, but they explained the question through presentation and guided the students. As the study progressed and the teachers noticed that their skill levels increased, they paid more attention to the students' thinking, tried to understand them, discovered new ideas that might emerge, and started focusing on students' mathematical thinking.

Keywords: noticing, modeling, teacher education, professional development, mathematics education.

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INTRODUCTION

Mathematics is a science that is sometimes directly reflected in our lives, and sometimes, we use it to make sense of the situations we encounter in everyday life (İncikabı, 2020). In addition to presenting a science so influential in our lives, it is possible to enable students to apply it in everyday life through effective mathematics teaching. Based on effective mathematics teaching, students can be brought to a level where they can quickly solve problems by using their mathematical thinking skills and developing skills such as creativity, critical thinking, communication, collaboration, problem-solving, reasoning, and estimation, which are referred to in the literature as 21st-century skills.

A learning approach has prevailed in mathematics education in which it is essential to teach students how to think and provide them with basic concepts and skills has been prevailed (Birgin & Öztürk, 2021; Bilgili & Çiltaş, 2019; National Council of Teachers of Mathematics [NCTM], 2000; Umay, 2007). One of the main goals of mathematics education is to educate people who can apply mathematics in daily life (Kaya, 2019). It is an accepted situation all over the world that mathematical modeling should be gained in raising individuals who can apply mathematics to problems in daily life and apply it to their lives (Bilgili & Çiltaş, 2022; Sağıroğlu & Karataş, 2018). Because mathematically expressing a situation in daily life, it is possible to express a bridge between mathematical modeling and daily life (Bilgili & Çiltaş, 2022; Bukova-Güzel, 2016; Ortiz & Dos Santos, 2011). Problems from daily life should be arranged so that students can develop and apply their reasoning, mathematical thinking, and estimation skills. These problems from daily life increase the importance of mathematical modeling (Kaiser & Schwarz, 2006).

Mathematical Modeling

The mathematical modeling process is when daily problems are solved, explained, and interpreted with the help of mathematical tools and mathematics. The mathematical modeling process is mental, and the product we obtain at the end of this process is a mathematical model (Sriraman & Lesh, 2006; Wess et al., 2021). The modeling process aims to predict problem situations, list possible situations, interpret them, analyze them through experiments, and create models in light of the obtained ones (Biembengut & Hein, 2010; Geiger et al., 2022; Lesh & Doerr, 2003). Modeling activities enable students to create models by thinking flexibly and creatively and questioning how they are modeled (Kaya, 2019).

Math teachers undoubtedly aim to raise individuals with mathematical modeling skills. In other words, the most crucial point for students' acquisition of modeling skills is that teachers experience modeling in the classroom with mathematical modeling activities (Sağıroğlu & Karataş, 2018). The mathematics classroom is a complex environment where multiple situations co-occur. Teachers should consider all stimuli and identify situations that require attention (Sherin et al., 2011). In addition, teachers should immediately notice the problems that students face during modeling activities and intervene. However, teachers cannot pay attention to all situations simultaneously; they should learn to filter this complexity and decide where to focus their attention and efforts in the teaching process (Sherin et al., 2011).

To provide the necessary skills in the renewed education approach, the responsibility and role of teachers is enormous. (National Council of Mathematics Teachers [NCTM], 2000; Umay, 2007). When implementing the modeling activity in the classroom, teachers should immediately respond to students' problems and provide a minimum amount of guidance for students to proceed correctly (Aydın et al., 2022; Leiss, 2007; Stender, 2016). Even if students do not directly express the problems they face, teachers should be able to notice the issues and use their professional knowledge to act in a situation-specific manner (Alwast & Vorhölter, 2021; Tekin-Sitrava et al., 2021).

Teachers' Noticing

The teacher who can notice plays an essential role in developing students' thinking and learning during the modeling activity by creating a productive learning environment (Türk & Baki, 2017). Noticing skill is a theoretical structure that reflects the teacher's observation, understanding, and interpretation of significant events in the classroom (Baş, 2013; Goldsmith & Seago, 2011; Jacobs et al., 2010; Santagata et al., 2021; Sherin & van Es, 2009; van Es, 2011). The modeling activities connect this theoretical structure between the teacher and the student. Thus, we can say that the modeling process is a system that allows the teacher to perceive how the student thinks.

The skill of noticing, which should be one of the professional competencies of every teacher, acts as a bridge between the student and the teacher in the teaching process. The teacher can reach as many students as he/she recognizes, guide the lesson, and make teaching more efficient. Noticing students in the use of modeling activities in the lesson is as essential and fundamental as the use of modeling activities in the classroom. Noticing skill, considered an essential component of specialization in teaching, significantly affects the quality of mathematics teaching (Dindyal et al., 2021; Jacobs et al., 2010; Schack et al., 2017). By focusing on students' thoughts, the teacher should decide what to deal with by being aware of students' mathematical thinking to understand where they make mistakes, where there is erroneous learning, what kind of thinking and reasoning they have in which situations, and reflect this in the teaching processes (Bastian et al., 2022; Biber & Özdemir, 2021; Taşdan, 2019). This reflects the ability to notice from teaching experience, an essential component of mathematics teaching (Sherin et al., 2011). The more the teacher perceives the students' thoughts, the more effectively the teacher manages the teaching process (Gürsoy, 2019; van Es & Sherin, 2010; van Es et al., 2022). Noticing does not only mean detecting students' mathematical thinking (van Es & Sherin, 2021). A teacher who can notice should also be able to make decisions for the next level stage according to students' understanding and thinking by paying attention to meaningful mathematical thinking (Krupa et al., 2017). In this way, a lasting and more effective teaching process takes place.

In line with the above, this study aims to investigate the changing noticing levels of mathematics teachers in practices carried out through modeling processes. The main research question of this research is: "How does the level of noticing skills of mathematics teachers change over time in the modeling process?". For all these reasons, this study, which dealt with modeling processes that enabled teachers to recognize students' mathematical thinking considerably, attempted to determine the change in teachers' noticing skill levels over time. A literature review found that very few studies examined the change in the level of noticing of mathematics teachers using the modeling activity. The findings of studies on teacher noticing skills show that teachers' focus on noticing skills helps them understand how students think and the structures they form in their minds during the solution process (Jacobs et al., 2010). There are many studies stating that noticing skills can be improved with appropriate professional development experiences by increasing the importance given to students' thinking (Goldsmith & Seago, 2011; Güner & Akyüz, 2017; Jacobs et al., 2010; Sherin & van Es, 2009; van Es, 2011; van Es & Sherin, 2008). However, these studies were conducted with pre-service teachers in the pre-service period and were not conducted with modeling activities before (Barnhart & van Es, 2015; Güner & Akyüz, 2017; Star & Strickland, 2008; van Es & Sherin, 2008; Walkoe, 2015). When the literature was examined, very few studies examined the changes in mathematics teachers' level of awareness during the implementation of the modeling activity. This study is essential in examining both mathematics teachers' use of modeling activities and the level of change in noticing skills, and it is thought that it will contribute to the literature, which has limited studies on these components. This study is significant both in terms of mathematics teachers' use of the modeling activity and in examining change in the level of noticing skills, and it will likely contribute to the literature, where there are few studies on these components.

METHOD

Research Design

This study aims to investigate the noticing abilities of mathematics teachers in the modeling process in-depth and the context of everyday life and to determine who, what, and how they notice and what their level of noticing is. To this end, the case study method, one of the qualitative research methods, was used to obtain broad, exhaustive, comprehensive, and in-depth data. The study is about seeing the details that make up an event, finding explanations for a situation, and evaluating a situation based on a study.

Participants and Procedure

The study sample consists of 7 elementary school math teachers working in a randomly selected state elementary school in a province in the Central Anatolia region of Turkey. Twelve math teachers worked in the elementary school where the study was conducted. At the beginning of the study, volunteer teachers from 2 different schools were informed about the purpose, scope, and process of the study, and the volunteerism of the participants was taken as a basis. The study continued with seven mathematics teachers who volunteered. Table 1 shows some demographic information of the participants. The teachers were coded as T1, T2 ..., T7.

Table 1. Some Demographic Information of Teachers

Teacher	Gender	Experience (Years)
T1	Male	11
T2	Male	8
T3	Female	12
T4	Male	9
T5	Female	8
T6	Female	11
T7	Male	10

As seen in Table 1, the participants' experiences are close to each other. None of the teachers has received any training in mathematical modeling. At the same time, there is also no training that the teachers have received beforehand. Two-week mathematical modeling training was given to the teachers before the application. This training explains the type of modeling activities and their classroom practices in detail.

Data Collection

The methods section must include the research design or the type of the study (cross-sectional, longitudinal, survey, experimental, ethnographic, etc.), the description of the sampling procedure (including the description of the population), or the selection of the study group, data collection instruments and procedures, data analysis, and the issues of validity, reliability, and ethics. More than one data collection instrument was used to ensure data diversity in the study, which investigated the changing skill levels of mathematics teachers in the process of applying mathematical modeling activities. Each data collection instrument used in the study was prepared to collect as detailed information as possible. The study used modeling activities, students' group worksheets, semi-structured interview sheets, observation notes, and video and audio recordings as data collection instruments.

Video and audio recordings

Video and audio recordings are used to collect data and identify missed situations to study them in detail in research. Researchers determine their impressions by re-watching the video recordings after the observation and identifying the situations they missed. Similarly, the researchers identified the situations and impressions they missed by listening to the audio recordings they made before and after the semi-structured interviews with the teachers.

Observation form

The situations observed by the researchers in the researchers in the classroom were transferred to the observation forms, taking into account the dialogs between the teacher and the student during the implementation of the modeling activities in the study. It was used to note the student's answers, how much the teacher noticed the answers given by the student in this process, and how the teacher gave feedback on what he/she did not notice.

Group worksheets
This is the work that students do together with their group mates to solve mathematical modeling problems in the math class. Group worksheets contain models, solutions, and mathematical reasoning created by each student in the group about the modeling activity they practiced in the lesson.

Mathematical modeling activities
The modeling questions, the validity and reliability of which were determined beforehand, were applied separately for each lesson during the 10-week implementation period of the study. They consisted of daily life problems that would make students active in the lesson and enable them to participate. All teachers applied the mathematical modeling questions at the 7th and 8th grade level. A pool of modeling activities consisting of 20 questions was presented to the teachers. The teachers decided together on the modeling activities that could be applied at the 7th and 8th grade levels. Then, each teacher implemented the classroom modeling activities in the order they determined according to their grade achievement levels.

Semi-Structured interview forms
During the study, semi-structured interview forms prepared before and after each mathematical modeling activity were applied to the teacher. During the application, the teacher conducting the semi-structured interview on modeling noted how much of the student's responses were, how he/she gave feedback, and what he/she thought about the students' responses.

Video and audio recordings were made of the course processes in which each teacher applied mathematical modeling activities in their practices, which lasted ten weeks, two hours per week. A total of 140 lessons hours were recorded on video and audio. During the application process of the teacher's mathematical modeling activity, lesson observation notes were filled. Semi-structured interviews were conducted with the teacher before and after applying the mathematical modeling activities by filling out. In this process, a semi-structured interview form and audio interviews were recorded. After the application, the solutions for the students' group activities were collected and used for data analysis. All the data obtained at the end of the studies were analyzed coordinately. The data sources mentioned in the research process enabled the identification and in-depth study of the teachers' level of noticing ability.

Data Analysis, Validity and Reliability
In analyzing the data, the descriptive analysis method was used to examine the qualitative data obtained and investigate the level of noticing skills of mathematics teachers' perceptual abilities in applying mathematical modeling activities. The primary purpose of this method is to present the research results in a summarized and interpreted form. In the first step of the research, the theoretical framework to investigate the level of noticing abilities within the context of the teachers' feedback and dialogues with the students during the implementation of the mathematical modeling activities and the teachers' responses in the semi-structured interviews was adopted with the framework developed by Estapa et al. (2018). Thus, it was determined which theme should be used for the data. The given framework was used when processing the data. The data obtained and processed were determined to define the findings. The findings defined during the interpretation of the results were explained and interpreted. In analyzing the data, teachers' noticing skills were examined using the framework developed by Estapa et al. (2018), which was in line with the purpose of the study. This framework's purpose is to reflect teachers' noticing skills expressly. Subsequently, teachers' noticing skills were determined using the framework developed by Van Es (2011).

Van Es (2011) examined noticing in two categories: What do teachers notice, and how do they notice it? The teachers then examined the categories of what they noticed and how they noticed it on four levels.

Four levels are determined by Van Es (2011) for the ability to notice; It is expressed as Level 1 (basic), Level 2 (mixed), Level 3 (focused), and Level 4 (extended). Van Es (2011) defined these levels to determine who, what, and how teachers notice. Van Es (2011) differentiated the teacher's notice and how he noticed the four levels in which he examined the components: Level 1 (basic), Level 2 (mixed), Level 3 (focused), and Level 4 (extended). This framework was used to determine teachers' noticing abilities in the mathematical modeling process and semi-structured interviews with teachers.

Estapa et al. (2018) developed the framework using the theoretical framework named by Van Es and Sherin (2008), and Jacobs et al. (2010) revealed and called the "Content and Stance Framework" in their work. The framework Estapa et al. (2018) developed is given in Figure 1.

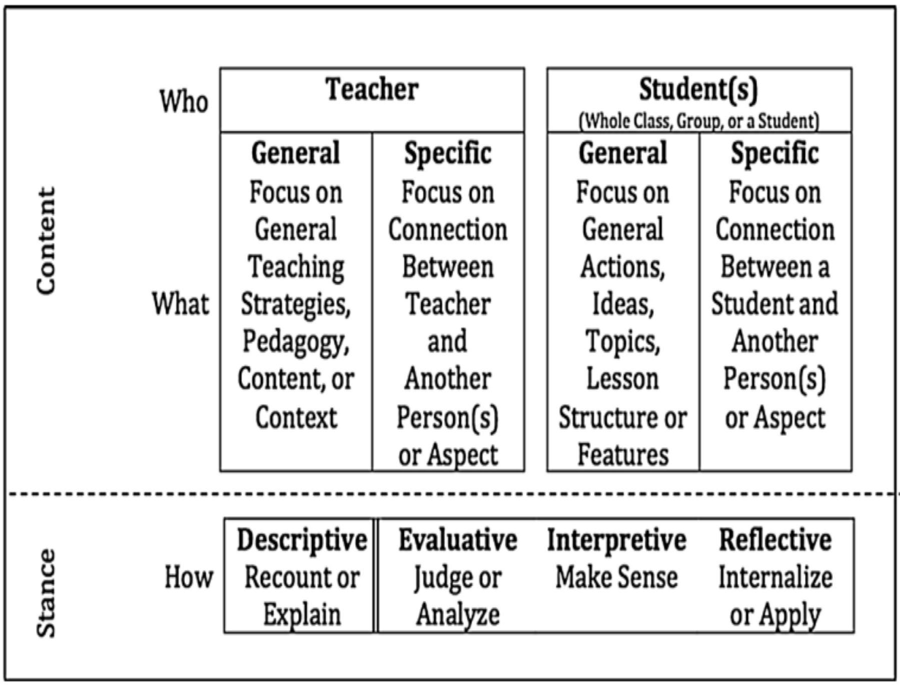

Figure1. Content and Stance Framework (Estapa et al., 2018)

The content analysis of the data obtained from the application was carried out in two ways. These analyses are retrospective and prospective. In analyzing the data consisting of the transcripts of the video recordings, the audio recordings of the semi-structured interviews with the teacher, the audio recordings of the student groups in the modeling process, the observation notes, and the students' solution papers in both analyses processes, Miles and Huberman's (1994) qualitative data analysis method was used, which consists of three stages, namely "data reduction," "showing the results" and "revealing and verifying the results," was used. At the data reduction stage, the raw data was extracted per the purpose of the study, and categories and themes were created by coding the data. In the representation of the data, the data was visualized with the help of tables or figures. The table used to display the data obtained by the observer during the implementation process can be found below.

Table 2. A Snippet from Data Analysis

Who				What		How				Teacher's Answers	Noticing Level
Teacher	Whole Class	Group	Student	General	Specific	Descriptive	Evaluative	Interpretive	Reflective		
T1	X			X		X					Level 1

As can be seen in Table 2, the data was primarily answered with the question "Who noticed?" and "What did he notice?" It was shown in the table within the scope of the components and recorded with the teacher's answers. The "How did he notice" levels of math teachers were determined by considering the repetition percentages of the codes identified for the general characteristics of each level. Then, the codes determined in each section are described in detail under the framework of the general characteristics of the level. The detailed table of level codes discussed will be improved over the weeks as additions are made to the existing codes. For this reason, the level table in the last week has the feature of a table containing all the codes that emerged in the study. Each week, the most common pattern observed by the teachers is identified, and the teacher's general "How did he notice?" level (descriptive, evaluative, interpretive, reflective) is determined. The decision process for the dominant level was based on the pattern observed in the teacher's codes at all levels, based on the fact that the number of codes observed at one level was 50% higher than the number of codes at other levels. Then, the determined level was shown with tables and supported by teacher responses.

In the prospective analysis, the interviews were continuously analyzed during the process, and as a result, alternative questions were produced without leaving the focus to make the interviews more effective (Cobb et al., 2003). In the retrospective analysis, all data collected at the end of the study were analyzed, and the model was found to be reliable and consistent (Cobb, 2000; Cobb et al. & Schauble, 2003; Steffe & Thompson, 2000). As a result of these conceptual analyses, the weekly tables were examined, and teachers' weekly noticing levels were decided.

To ensure the reliability of the data analysis, the data were independently coded by the second researcher and another mathematics educator (external observer) who is an expert in the field of teacher knowledge. The number of "agreement" and "disagreement" for the coding made by the two researchers was determined. The reliability coefficients of the researchers were used with Miles and Huberman's (1994) formula "Reliability = (Agreement) / [(Agreement) + (Disagreement)]," and the reliability of the data analysis was found to be 89%.

FINDINGS

All the findings obtained in the study are given in detail in the form of themes. These themes consist of three categories: "Who did the teacher notice?" "What did the teacher notice?" and "How did the teacher notice?". In the first week of the exercise, the T4 teacher was observed. The observation sheet with the topics "Who did the teacher notice?", "What did the teacher notice?" and "How did the teacher notice?" is listed in Table 3.

Table 3. Observation Form of T4 During the First Week

Who				What		How				Teacher's Answers	Noticing Level
Teacher	Whole Class	Group	Student	General	Specific	Descriptive	Evaluative	Interpretive	Reflective		
T4		X		X			X				Level 2

While the T4 teacher was practicing in the classroom, he generally focused on the solution results of the groups and did not realize the students' mathematical thinking. The semi-structured interviews with the teacher showed that the teacher focused more on the groups. This shows that the component "who did the teacher notice" in the table above is the group. When examining what the teacher noticed, the semi-structured interviews with the teacher, and the dialogs between the teacher and the student during the application, it was found that the teacher noticed the class in general. While creating the tables, the percentage of each teacher's "how did the teacher notice" component was calculated during the application, and semi-structured interviews with teachers and observations were considered. The codes and percentages identified by the teachers when using the mathematical modeling activities are shown in the tables for each week. The level of noticing determined by the teachers in the first week during the application process of mathematical modeling activities is shown in Table 4.

Table 4. Teachers' Noticing Levels Determined in The First Week

Noticing Levels	Teachers						
	T1	T2	T3	T4	T5	T6	T7
Level 1	X	X	X			X	X
Level 2				X	X		
Level 3							
Level 4							

As shown in Table 4, it was determined that 5 of the seven math teachers observed in the first week were at Level 1, and the noticing level of the other two teachers was at Level 2. In determining the teachers' noticing levels in the table, the percentage of each teacher's "How did the teacher notice" component during the application was considered, and semi-structured interviews and observations with the teachers were considered. The "How did he/she notice" levels of the math teachers were determined by taking into account the repetition percentages of the codes determined for the general characteristics of each level. In the first week, the number of repetitions and level percentages of the codes determined how teachers noticed during practice were calculated. For example, in the T4 teacher, during the application of the descriptive level codes, It was observed that the student described the student's operations in the modeling process two times without comment, inconsistency in the teacher's comments, and the student's solution approaches three times, the teacher rephrased the question 5 times in a different language so that the student could understand the modeling activity, and it was seen that he ignored the answers to the questions asked to the student once. From the codes of the evaluative level codes of the T4 teacher, Focusing on the correctness or incorrectness of student solutions five times, assuming an understanding of the concept if the student's solution is correct, and not understanding the concept if the student's solution is incorrect six times, focusing on the models they create at the end of the process, and not on the students' thinking during the modeling process, four times. In contrast, the teacher supports his explanations and mentions the students and the specific moments he observed during the process. It was observed that he did not deepen his comments and repeated the situations he observed three times. From the T4 teacher's interpretive level codes, It was observed that he never focused on the student's mathematical thinking; twice, it was observed that he paid attention to the models that the students created during the modeling process, and once he gave feedback through focusing on the students' group discussions and noticing the misconceptions. It was observed that it was never seen that relying on the teachers' observations, examining critical situations that occur, and considering the details of the situations that students examine to conclude make inferences about mathematical understanding and thinking processes. For teacher T4, no observations were made in the codes at the reflective level. The codes at this level are: the teacher extends his analysis to understand the students thinking, the teacher supports his studies by basing them on various explanations and interpretations, the teacher links his analysis of the student's thoughts to particular approaches, the teacher offers an alternative teaching approach that is consistent with the analyzes made in this process, are never repeated by the teacher. Considering these codes together, we find that the number of repetitions of the codes at the descriptive level is 11, the codes at the evaluative level are 18, the interpretive level is 3, and the number of codes at the reflective level is 0. Then, the codes obtained in each section are described in detail within the general characteristics of each level. If we break down these obtained code numbers into percentages, a total of 11 out of 32 are descriptive, and the percentage equivalent of this expression is 34%; likewise, the number of codes at the evaluative level is 18 out of 32, which corresponds to 56% as a percentage, and the number of codes at interpretive level is 3 out of 32. times, and it was found that this corresponds to 6% as a percentage. The number of codes at the evaluative level outweighs the total number. It can be seen that the number of codes at the evaluative level accounts for 56%, i.e., more than 50% of the total number of codes. Based on these, it was concluded that the T4 teacher's level of noticing was appropriate for Level 2. These calculations and codes can be found in Table 5, which also shows the percentages and codes calculated for all teachers per week.

Table 5. The Number of Repetitions of the Codes Determined How the Teachers Noticed in the First Week of Practice and the Percentage of Codes based on Levels

Levels	**T1**	**T2**	**T3**	**T4**	**T5**	**T6**	**T7**
Reflective	0	0	0	0	0	0	0
The teacher expands their analysis to make sense of student thinking.	0	0	0	0	0	0	0
The teacher supports her/his studies by basing them on different explanations and interpretations.	0	0	0	0	0	0	0
The teacher relates their analysis of student ideas to specific approaches.	0	0	0	0	0	0	0
The teacher presents an alternative teaching approach in line with the analyses made in this process.	0	0	0	0	0	0	0
Interpretive	0	0	0	3/32 %6	0	0	0
Focus on students' mathematical thinking.	0	0	0	0	0	0	0
Paying attention to the models that the students created during the modeling process.	0	0	0	2	0	0	0
Give feedback by focusing on the students' group discussions and noticing misconceptions.	0	0	0	1	0	0	0
Reasoning on teacher observations, examining critical situations, and considering details of situations students examine to make inferences about mathematical understanding and thinking processes.	0	0	0	0	0	0	0
Evaluative	14/40 %35	16/40 %40	12/38 %32	18/32 %56	16/27 %60	8/24 %33	2/28 %7
Focus on the correctness or incorrectness of student solutions.	6	4	3	5	4	2	2
Assuming that the concept is understood if the student's solution is correct and that the concept is not understood if the student's solution is incorrect.	2	3	4	6	3	1	0
They should focus on the models they create at the end of the process, not on students' thinking during the modeling process.	4	4	3	4	4	3	0
Being inconsistent in deepening comments and expressing observed situations, even though the teacher supports her/his explanations and mentions specific moments and students in the process	2	5	2	3	5	2	0
Descriptive	26/40 %65	24/40 %60	26/38 %68	11/32 %34	11/27 %40	16/24 %67	26/28 %93

The student's actions during the modeling process are described without comment.	7	6	6	2	3	5	5
Teacher comments and student solution approaches are incompatible.	5	5	7	3	5	3	8
Re-phrase the question in a different language so the student understands the modeling activity.	8	6	7	5	2	5	4
Ignoring the answers to the questions posed to the student.	6	7	6	1	1	3	9

Mathematical noticing skill levels of teachers were calculated every week, and the findings were obtained from the dialogues of the teachers with the students during the implementation process of the mathematical modeling activities, the observations, and the semi-structured interviews with the teachers before and after the application. The section on semi-structured interviews with the teacher, the dialogue section between the teacher and students during the application process, and their explanations are reproduced below.

A modeling activity (see Figure 2) and a section of the dialogue between the teacher and the students are among the findings we obtained during the first week of T4. The teacher applies a mathematical modeling activity called "Tariff" as follows.

Tariff

A GSM operator offers two different tariffs, named A and B, for its customers; Tariff A is charged as 75 Penny per minute with a monthly fixed fee of 10 TL. Tariff B gives 10 minutes of talk time for a fixed fee of 17 TL per month. Apart from that, each minute is charged over 50 Penny. At the end of any interview period, try to find a mathematical expression and a graphic representation to express your debt for both tariffs. Decide which tariff is more economical in which case.

Note: For conversations lasting less than 1 minute, the 1-minute tariff is still valid.

Figure 2. A Mathematical Modeling Activity Called "Tariff"

...

S1: Teacher, does it pay when the fixed fee of 10 TL exceeds, or is it within the fee?

T4: The price he will pay even if he does not talk about a fixed fee of 10 TL. Each time he speaks, the amount will be multiplied by 0.75 and calculated accordingly. For example, if he spoke for 1 minute, you will receive 10+0.75. However, be careful; someone is TL, and someone is a penny, and when solving them, they must be from the same unit. If you say 1000 pennies for 10 TL, you will continue by adding 0.75 pence. If you talk for 2 minutes, it will be 1150, and so on. For example, who has the advantage when he does not speak?

S2: which is 10 TL.

T4: Well, at what minute will these packages not be superior to each other? How many minutes will it equal? Is there a time when you would say that it does not matter to us whether we choose A or B?

S1: We found 8.

S3: Again, we are giving 1700 penny for tariff B. If we write 8 for tariff A, It is 1600 pennies. Then, tariff A is still advantageous.

T4: Then you can try 9, not 8.

S4: We paid 16.75 for A. It's 17 for B.

T4: Did you investigate for the 10th minute?

S2: Tariff B is still at 17. Tariff A is 17.50.

T4: How would you comment then?

S1: In the 9th minute, A has the advantage. 10th minute B is advantageous.

T4: Exactly. Then the A package has the advantage until the one who does not talk and talks for 9 minutes. The B package becomes advantageous when I speak for 10 minutes. Well, one more question

for the B tariff: Is there a continuity of the B package after the 10th minute? If I talk in 1 hour, is the B package still advantageous?
S3: Let's see. It is 2500 for tariff A, for B…
T4: Can't we say that B will always be more advantageous after the 10th minute since B will always increase with lower wages?
…
Examination of the dialog between the teacher and the group revealed that the teacher assumed that the solution results were understood when they were correct and not incorrect. In cases where the students did not understand the modeling activity, it was observed that they rephrased the question in a different language so that they could understand it. The teacher did not focus on the student's mathematical reasoning but gave them hints during the modeling activity and guided them to the conclusions. During the modeling activity, it was found that students focused on the models they had created at the end of the process rather than on their thinking. This showed that the T4 teacher noticed the student groups in general during the application process in year 1.
A section of semi-structured interviews with the T4 teacher before and after the application is below.
…
Researcher: What kind of path will you follow in the implementation process of modeling activities?
T4: I will inform the students about the modeling activity I will apply. I will make them think again by explaining it to those who have questions or do not understand, and I will get their feedback. After working individually, I will group the class so that they learn in the group and think by collaborating.
Researcher: How did you go about implementing the modeling activity in the classroom?
T4: I started by explaining the question to the students. I gave them time to work individually. Then, I divided them into groups. In turn, I gave slight hints to each group at different times and guided them to the solution.
…
When examining the above dialog section, it was found that the teacher checked the results by giving feedback according to the models created by the student groups and the solution results during the application. The teacher asked the students who knew little more straightforward questions and ensured they participated in the lesson and the solution process. In general, the teacher evaluated the groups and the models created due to the modeling activity with great attention. Although the teacher talked about the specific moments and students he observed while reinforcing his explanations, there were inconsistencies in the deepening of his comments and the expression of the situations he observed. These observed situations also show that the teacher is at the evaluative level. Considering all this data, it was concluded that when the teacher noticed whom he noticed, he noticed the groups; when he examined what he noticed, it was general, and when he examined how he noticed, it was concluded that he was an evaluator. All this shows that the teacher's noticing level is at level 2. The following Table 6 was created when the noticing level of the observed teachers was examined during the implementation of the modeling activities during the ten weeks.

Table 6. Throughout the Ten-Week Implementation Process, Teachers' Noticing Levels Changed

	Weeks	**First**	**Second**	**Third**	**Fourth**	**Fifth**
LEVELS	**Level 1**	T1, T2, T3, T6, T7	T1, T3, T6, T7	T1, T3, T7	T7	T7
	Level 2	T4, T5	T2, T4, T5	T2, T5, T6	T1, T2, T3, T5, T6	T1, T2, T3, T6
	Level 3			T4	T4	T4, T5
	Level 4					
	Weeks	**Sixth**	**Seventh**	**Eighth**	**Ninth**	**Tenth**
LEVELS	**Level 1**	T7	T7	T7	T7	T7
	Level 2	T1, T3	T1, T3	T1, T3	T1, T3	
	Level 3	T2, T4, T5, T6	T2, T5, T6	T2, T5, T6	T6	T1, T3

Level 4	T4	T4	T2, T4, T5	T2, T4, T5, T6

When Table 6 was examined, it was found that 6 out of 7 teachers showed an increased level of noticing skills during the mathematical modeling activities. The levels progress towards Levels 1, 2, 3, and 4. One of the observed teachers started with level 1 and progressed to level 1. An increase was observed in the skill levels of the other six teachers. The increase in teachers' noticing skills was reached by analyzing the data obtained during the application and the interviews with the teachers before and after the application, calculating the codes corresponding to each behavior, and determining the percentages.

DISCUSSION AND CONCLUSION

In this study, the levels of noticing skills of mathematics teachers during the implementation of the mathematical modeling activity were determined using the framework developed by Estapa et al. (2018). Subsequently, teachers' noticing skills were determined using the framework developed by van Es (2011). The mathematical modeling activities used in changing the level of teachers' noticing skills effectively revealed students' mathematical thinking and generated different mathematical thinking in group work. They questioned students' understanding and mathematical thinking. About the levels in the frameworks used during the research process over time, six out of seven teachers observed an increase. During the application of the modeling activity, no progress was observed since the teacher with code T7, who did not show an increase in his noticing skill, was closed to development. Although the activity applied was a mathematical modeling activity, he overlooked the students' interpretations and mathematical reasoning, taking into account the result-oriented solutions. There were deficits in the context of communication with the students. The hypothesized factors that are thought to impact the increase in teachers' noticing skills during the implementation phase of the modeling activities are as follows. The semi-structured interviews with the teacher before and after the application revealed that the teacher's self-monitoring by reflecting on the implementation process and paying attention to behaviors that they perceive as incomplete or incorrect in other applications, as well as focusing more on the student's mathematical thinking and trying to notice it them had an impact. Studies in the literature indicate that teachers' noticing skills can change and develop with long-term practice and training (Özdemir et al., 2018; Star & Strickland, 2007). Güner and Akyüz (2017) found that pre-service teachers' noticing skills can be improved by using the lesson study method. Looking at these studies, teachers' long-term professional development training can be increased, and the noticing skills of teachers can be enhanced by providing the necessary information to recognize students' thinking. The mathematical modeling activities used in the study also play an essential role in the emergence of the teacher's ability to notice. With modeling activities, teachers can easily observe the students and design the teaching situations according to the student's mathematical thinking. When using the modeling activities, the teacher can intervene, when necessary, by noticing the students' thinking, the conflicts that arise when working in cooperation with the student groups, and the student's mathematical thinking by observing more closely the misconceptions of the students. The classroom environment will be created with activities requiring students to express their thoughts and discuss the concepts rather than focusing on the results. Having students perform on paper will also contribute positively to students (Straker, 1993). Class discussions and group presentations following the applied mathematical modeling activities also contribute to the development of the mathematical language that students use to express themselves. Ulusoy and Çakıroğlu (2018) stated in their study with prospective teachers that through the video analysis and interviews focusing on students' mathematical thinking, they could recognize students' mathematical thinking by analyzing it more deeply.

In this study, which was conducted with teachers, it was observed that students' mathematical thinking and ability to recognize the situations encountered were generally low and insufficient initially, but their noticing skills increased over time. The results of similar studies in the literature also show that teachers and prospective teachers have insufficient ability to recognize students' mathematical thinking (Güner

& Akyüz, 2017; Roth McDuffie et al., 2014; Sherin & Han, 2004; Sherin & van Es, 2005, 2009; van Es, 2011; van Es & Sherin, 2008).

In the study, it was found that the result on the level of noticing skills of math teachers was at Level 1 or Level 2, like the level of noticing skill in general, and there were no teachers with Level 3 and Level 4 awareness at the beginning of the study. After the investigation, it was found that noticing skill improved and reached level 2, Level 3, and level 4. It was found that 2 of the improved teachers moved up to level 3, 4 of them to level 4, and 1 of them started at level 1 and remained at level 1. As a result of the investigation, it can be concluded that the teachers lack of teachers' noticing skills. It is seen that this result is similar to the results of similar studies in the literature (Güner & Akyüz, 2017; Star & Strickland, 2008).

In examining what teachers noticed during the implementation process, it was found that although the teacher generally noticed the whole class, noticing evolved as noticing the mathematical thinking of groups or individuals. Similar results were obtained in other studies. At the beginning of the study, although the teachers commented on the situations they noticed in semi-structured interviews, it was found that they could not give any details of the situations they noticed, and they conveyed the situation superficially. They found that they focused on the relationship between the students' thoughts. The fact that the teachers tried to reveal their mathematical thinking by focusing on students' thoughts showed they were aware of the importance of students' thinking. In terms of how the teachers noticed, they were found to use more evaluative and descriptive approaches at the beginning while taking an interpretive and reflective approach as the process progressed. When the literature, analytical attitudes, and explanations of teachers with lower noticing skills are examined, it is shown that they are primarily descriptive and evaluative. According to the analyzes and investigations in similar studies, it was found that teachers or prospective teachers' explanations are usually descriptive and evaluative at the beginning (Güner & Akyüz, 2017; Roth McDuffie et al., 2014; Sherin & Han, 2004; Sherin & van Es, 2005, 2009; van Es, 2011; van Es & Sherin, 2008).

As the perceptual skills of the teachers' noticing skills, the students who could not understand the question were noticed, the necessary hints and feedback were given, and an increase in the correct meaning of the problem and the effectiveness of the teaching was observed. It was observed that students' motivation increased to a certain extent due to the feeling of being noticed, their communication with teachers strengthened, and students did not hesitate to express themselves. They were encouraged to express their mathematical thoughts. Based on the interviews with the teachers at the end of the study, it was stated by the teachers that the students who were encouraged increased their interest in the lesson and started to express their mathematical thoughts and interpretations more, and this improved teacher-student relations in the classroom.

Since the modeling questions used in the study were problems that students could encounter daily, the teacher who implemented the mathematical modeling activity stated during the interviews that the modeling questions facilitated students' understanding of the question. It was observed by the teacher that the students approached the solution process by seeing the connection of the modeling questions with daily life as a problem situation they encounter. İn fact, it was observed that a student who understood the question and could express himself quickly in the solution process made various comments on the models he had developed during the modeling process instead of finding only one result in the modeling process. It was observed that this process had an essential place in the teacher's noticing of the student.

It was observed that the students, who felt responsible when they perceived the teachers, checked the modeling process in different ways in the modeling process by asking whether they found values close to reality when they created the model and whether they were actively in the group by discussing the suitability of the mathematical thinking or models they created during the solution process without the support of the teachers and making self-assessments. The teachers noticed each student, and it was found that even the most passive student actively participated in the lessons at the end of the process, and his interest and pleasure in the lessons increased.

As the research continued for ten weeks, during the semi-structured interviews conducted in the process, teachers reflected on the situations that occurred during the implementation process, and the differences in teachers' courses and the approaches to students were readily apparent. While the teachers guided the students by explaining the question through a presentation, without allowing the students to understand and interpret the question at the beginning of the research, as the investigation progressed and the teacher noticing level increased, the teachers paid more attention to the students' thinking and tried to understand them, to discover new ideas that might arise and to help the students think mathematically., Furthermore during the research, it was found that the mathematical thinking that teachers did not hear, see, overlook or ignore during lesson, both in the semi-structured interviews and when using and their questioning skills of the students increased. As their noticing skills improved, the teachers paid more attention to the students' mistakes, misunderstandings, or different approaches, questioned them, and gave their feedback by trying to identify the underlying cause. At the end of the lesson, the teachers, whose noticing level was developed, gave clear and summarized explanations to clear the question marks in the students' minds. During the semi-structured interviews with the teachers before and after the application, the teachers' focus on the teaching process and thinking about the process in detail helped raise the teachers' noticing skills. İn addition, the teachers increased their noticing skill levels by increasing their perceptual skills in the implementation phase of the modeling activity. In the implementation phase of the modeling activity, they noticed the students' mathematical thinking and gave feedback according to their level and situation. In the interviews conducted due to the implementation, teachers stated that when they felt respected and cared for, their interest and motivation in the lesson increased, and communication between teachers and students strengthened. Teachers were prepared for any situation that might occur during the lesson. The teachers, whose noticing skills have improved due to the study, stated that although they were aware of the situations in which the students had difficulties before the study, they noticed other situations that they could not notice with this study. According to the results of the studies in the literature, it is expected that teachers who have developed noticing skills will have students apply mathematical modeling activities after the end of the research and thus contribute to the development of students' mathematical thinking by making comments, and at the same time, with group work in mathematical modeling activities, students will participate more actively in the lesson by integrating their reflections in daily life with the modeling activity in the lesson.

They indicated that they would contribute to developing students' problem-solving skills. The teachers, whose noticing skills developed at the end of the research and who recognized the contribution of this study to the teaching process during the 10-week practice, indicated that they would recognize students' mathematical thinking and guide the process accordingly by using what they learned during the research process.

It is believed that the suggestions developed based on the data obtained in this study regarding the development of elementary school mathematics teachers related to the level of noticing in the modeling process will guide future studies. Teachers can receive in-service training to improve their noticing skills. In this training, the standard course process of the teachers can be videotaped, and teachers' self-assessments and self-evaluations can be supported after the lesson. Noticing skills can be offered as an elective course at universities to improve the perceptual skills of pre-service teachers' ability to notice. After training, teacher candidates' noticing skills can be observed step by step in appropriate environments. The development of mathematics teachers' level of noticing skills during the implementation of modeling activities can be recorded on video. Teachers can be observed, and their professional development can be supported by asking them to self-evaluate.

CONCLUSION

As the research continued for ten weeks, during the semi-structured interviews conducted during the process, as the teachers reflected and reviewed the situations during the implementation process, differences were readily observed in the teachers' teaching and approach to students. At the beginning of the study, the teachers did not allow students to understand and interpret the question, but they explained the question through presentation and guided the students; however, as the study progressed and the teacher's noticing skill level increased, the teachers paid more attention to the students' thinking, tried to understand them, started to discover new ideas that might emerge, and started to focus on students' mathematical thinking.

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